NIOSE DRIVEN OSCILLATION OF SOLUTIONS OF STOCHASTIC OPTIMAL PROBLEMS WITH A CONTROLLED TIME DELAY TARGET

Atonuje, A. O.
Department of Mathematics & Computer Science, Delta State University, Abraka.

Email Address: austino412@yahoo.com

ABSTRACT:
The paper studies how an Ito-type noise perturbation influences the creation, existence and destruction of oscillation in solutions of a stochastic optimal time lag control system with a delayed target. We establish that if the noise is absent under certain conditions, the system can admit a non-oscillatory solution. This cannot happen in the presence of noise irrespective of the magnitude of the time lags. We contrast the oscillatory behaviour of the system with that of a comparable classical delay differential system to justify the effectiveness of our results.

Keywords: Noise perturbation, stochastic delay optimal control system, oscillation, classical delay differential system, time lag.

INTRODUCTION

In modern technology, physical processes are usually controllable, that is, they depend in some way on our will. Such systems often involve non-negligible time delays between any particular incident in the behaviour of the quantities being controlled and the result of the operation of the controlling system due to the incident. Hence processes of this kind are described using delay differential equations (DDEs).

The significance of delay differential equations lies on their ability to describe processes with memory or after-effects. They are applicable in various branches of technology, economics, biology and medical sciences and this has caused mathematicians to study them with increasing interest (Gopalsamy,1992; Agwo, 1999). In the last two decades, an enormous number of paper articles had been devoted to delay differential equations.

In recent years, there have been much research activities concerning oscillation theory of DDEs. For instance, Elabbasy *et al* (2000) considered the DDE with positive and negative coefficients of the form

$$x^{\prime}(t)+P(t)x(t-\sigma)-Q(t-r)=0 \qquad (1.1)$$

and its more general form

$$x^{\prime}(t)+\sum_{i=1}^{n}P_i(t)x(t-\sigma_i)-\sum_{j=1}^{n}Q_j(t)x(t-r_j)=0 \qquad (1.2)$$

and proved that

$$h_1 : P, Q \in C([t_0,\infty), \Re^+), \sigma, r \in [0,\infty), r \le \sigma$$

$$h_2 : P(t) \ge Q(t+r-\sigma) \; for \; t \ge t_0 + \sigma - r$$

$$h_3 : \int_{t-\sigma}^{t-r} Q(s)ds \le 1 \; for \; t \ge t_0 + \sigma$$

$$h_4 : \int_{t}^{t+\sigma} R(s)ds > 0 \; for \; t > t_0 \; for \; some \; t_0 > 0$$

$$where \; R(t) = P(t) - Q(t+r-\sigma)$$

$$h_5 : \int_{t_0}^{\infty} R(t) In\left[e\int_{t}^{t+\sigma} R(s)ds\right]dt = \infty$$

Are necessary for oscillation of all solutions of Equ. (1.1). Other authors which include Ahmed (2003), Agwo (1999), Li (1996) have introduced different new techniques to obtain both necessary and sufficient conditions for the oscillation of solutions of different forms of DDEs.

In spite of all these efforts, it appears that the influence of noise perturbation of Ito type on the creation, existence and destruction of oscillation in solutions of vector DDEs is presently quite little. The first paper on the contribution of noise perturbation to oscillation of feedback processes was written by Appleby and Buckwar (2005). Other authors include Appleby and Kelly (2004), Atonuje (2010) and some of their references therein.

In this paper, we employed the formalism of Appleby and Buckwar (2005) as well as a transformation technique by Lisei (2001) and certain classical results in the theory of oscillation in DDEs to study the influence of Ito type noise perturbation on the oscillation in solutions of a stochastic optimal time lag control system whose state at time t is given by a linear stochastic DDEs

$$\left.\begin{aligned} dX(t) &= \left[\sum_{i=1}^{n} A_{ij}(t)X(t-r_{ij}) + \int_{t_0}^{t} K(\gamma,t)X(\gamma)d\gamma + A(t)u(t)\right]dt + \sigma X(t)dB(t) \\ X(t) &= \psi(t), \quad t \in I[\alpha,t_0], \quad \psi(t)\in\Phi \end{aligned}\right\} \quad (1.3)$$

Where $r_{ij,} \quad i,j = 1,2,\ldots\ldots\ldots\ldots,n$ are time lags such that $r_{11} < r_{12} < \ldots\ldots\ldots\ldots < r_{1n} \quad K(\gamma,t)$ $A_{ij}(t),\ i,j = 1,2,3,\ldots\ldots\ldots\ldots,n$ are given n x n continuous matrix functions, A(t) is a given continuous n x 1 matrix, X(t) is an n-dimensional vector which describes the state of the control system, σ is the noise scaling parameter and $\{B(t)\}_{t\geq 0}$ is a standard one dimensional Brownian motion on a complete probability space $(\Omega, F, \{F_t\}_{t\geq 0}, P)$ with a natural filtration $\{F_t\}_{t\geq 0}$, u(t) is an n-dimensional column vector controlling the motion of the system.

The initial function $\psi(t)\in C(I[\alpha,t_0],G)$, where G is a complex and compact set in an n-dimensional Euclidean space E^n and $\psi\in\Phi$, the set of all admissible initial functions.

PRELIMINARY NOTES:

By physical limitation of optimal control system, we assume that X is controlled by the n-dimensional vector $u(t) = (u_1(t), u_2(t), \ldots\ldots\ldots\ldots\ldots u_n(t))$ called control vectors. Let G be a compact region in an n-dimensional Euclidean space E^n which has the origin as an interior point. Let $I[t_1,t_2]\ where\ t_0 \leq t_1$ be a compact interval, then the range of all u(t) in G defined for $I[t_0,\bar{t}]\ where\ \bar{t} = [t_1,t_2]$ is called admissible and we denote by U, the set of all admissible control vectors.

In a control system X, the allowable or admissible initial functions $\psi(t)$ are restricted and we denote by Φ, the set of all admissible initial function

$$\Phi = \{\psi(t)\in C(I[\alpha,t_0],G),\ \psi(t)\ admissible\},\ where\ |\alpha|$$

is sufficiently large. We shall call $(u(t),\psi(t))$ an admissible pair or policy. We shall often contrast the oscillatory properties of the stochastic delay optimal control equation (1.3) with those of the comparable deterministic delay optimal system

$$x^{|}(t) = \sum_{i=1}^{n} A_{ij}(t)x(t-r_{ij}) + \int_{t_0}^{t} K(\gamma,t)x(\gamma)d\gamma + A(t)u(t) \quad (2.1)$$

With the same initial function ψ.

Definition 1:

By solution of the classical DDE (2.1), we mean a continuous vector function

$$x(t) = x(t;t_0,\Phi,u)\in C(I_\beta,G),\ t_0 < \beta \leq \gamma$$
$$such\ that\ x(t;t_0,\Phi,u) = \psi(t)\ for\ t\in I_{t_0}$$

which satisfies (2.1) as well as the initial condition $x(t) = \psi(t),\ t\in I[\alpha,t_0],\ \psi\in\Phi$ for sufficiently large $|\alpha|$. Also by solution of the stochastic delay optimal control system (SDOCS) (1.3), we mean an $\Re^n-$ valued function

$$X(t) = (X_1(t), X_2(t), \ldots\ldots\ldots X_n(t))\in C(I_{\beta,}G)$$

which is a measurable sample continuous process satisfying equation (1.3) almost surely together with its initial function. It is unique if any other solution Y(t) of (1.3) is indistinguishable from it. That is,

$$P(X(t) = Y(t),\ \alpha \leq t \leq \bar{t}) = 1.$$

Definition 2:

Recall that for scalar systems, a non-trivial solution x(t) of a classical DDE is said to be oscillatory if and only if if has arbitrarily large zeros for $t \geq t_0$, that is if there exists a a sequence $\{t_n : x(t_n) = 0\}\ of\ x(t)$ such that

$\underset{n\to\infty}{Limit}\, t_n = +\infty$. Otherwise, x(t) is said to be non-oscillatory. Gopalsamy (1987) has the following for vector systems:

A real valued differentiable function u defined on a half line $[t_0, \infty)$ is said to be oscillatory if there exists a sequence $\{t_m\} \to \infty \; as \; m \to \infty$ such that

$t_m \in (t_0, \infty) \; and \; u(t_m)\dot{u}(t_m) = 0 \; for \; each \; t_m \in (t_0, \infty) \; where \; \dot{u}(t) = du/dt \; at \; t_m.$

u is said to be non-oscillatory on $[t_0, \infty)$ if there exists $t^* > t_0$ such that $u(t)\dot{u}(t) \neq 0$ for $t > t^*$. We now apply this to (2.1) as follows:

Definition 3:

The solution $x(t) = (x_1(t), x_2(t), \ldots\ldots\ldots x_n(t))$ defined on $I[I_\beta, G] \subset [t_0, \infty), t_0 < \beta < \gamma$ with differentiable components is said to be oscillatory if at least one component of x is oscillatory in the sense of definition (2ii). A vector $x : [t_0, \infty) \to \Re^n$ with differentiable components is said to be non-oscillatory if x is non-oscillatory in the sense of definition (2ii).

We have similar definition for stochastic processes given in Appleby and Buckwar (2005).

Definition 4:

A non-trivial continuous function $f : [t_0, \infty) \to \Re$ is oscillatory if the set

$W_f = \{t \geq 0 : f(t) = 0 \;\} \; satisfies \; Sup W_f = \infty.$

A function which is not oscillatory is called non-oscillatory. We now extend the above to the solution of equation (1.3) in the following intuitive manner:

A solution $\{X(t,w)\}_{t \geq t_0}$ defined on a probability space (Ω, F, P) with continuous sample paths is said to be almost surely (a.s) oscillatory if there exists $\Omega^* \subseteq \Omega \; with \; P[\Omega^*] = 1$ such that for all $w \in \Omega^*$, the path X(.,w) is oscillatory.

It is well known that oscillation in solutions of delay differential equations is caused by the presence of a sufficiently large delay Ladas (1979). We proposed to establish that under certain conditions, oscillation is stimulated by the addition of a multiplicative noise perturbation in a previously non-oscillatory DDE without noise. Our technique involves decomposing the solution of the SDOCS (1.3) into a conjugation relation with the solution of a classical vector DDE of the form.

$$Z^{|}(t) = -\sum_{i=1}^{n} P_{ij}(t) Z_j(t - r_{ij}) \qquad (2.2)$$

Where $P_{ij}(t), \; i, j = 1, 2, \ldots\ldots\ldots\ldots, n, \quad t > t_0$.

Here the P_s depend on the increments of a standard Brownian motion and can be expressed fully in the form

$$P_{ij}(t) = \begin{cases} -k\exp\left(-\left(A - \frac{\sigma^2}{2}\right)\right) r_{ij} \exp(-\sigma(B(t) - B(t - r_{ij}))), & t > t^* \\ -k\exp\left(-\left(A - \frac{\sigma^2}{2}\right)\right) t - \sigma B(t), & t \leq t^* \end{cases} \qquad (2.3)$$

Where

$t^* = \inf\{t > 0 : t - r_{ij}\} \; such \; that \; t \to r_{ij} \geq 0 \; for \; all \; t > t^*$

almost surely. If the increments are sufficiently large, the P_s, can induce oscillation in the solutions of equation (2.2). We shall then invoke, on a path-wise basis, some extensive existing results in the theory of oscillation of delay differential equations (DDEs) which apply directly to equation (2.2), that is, for each $w \in \Omega$. The following concerning oscillatory solutions is a special case of the result found in Ladas (1979).

PROPOSITION 1:

Let $r_{ij} \in (0, \infty) \; and \; P_{ij} : [0, \infty) \to [0, \infty)$ be continuous such that

$$\underset{t\to\infty}{Limit} \inf \int_{t-r_{ij}}^{t} P_{ij}(s)ds > \frac{1}{e} \quad and \; \underset{t\to\infty}{Limit} \inf \int_{t-\frac{r_{ij}}{2}}^{t} P_{ij}(s)ds > 0 \qquad (2.6)$$

Then every solution of the equation

$$x^{|}(t) = -\sum_{i=1}^{n} P_{ij}(t) x(t - r_{ij}) \qquad (2.7)$$

oscillates.

We also have results concerning non-oscillation. The following is a special case of the result found in Tang and Yu [2000]. It is a general comparison theorem in the case when

$P_{ij}(t)-\frac{1}{r_{ij}e}$ is

$$\underset{t\to\infty}{Limit}\inf \int_{t-r_{ij}}^{t}\sum_{i=1}^{n}P_{ij}(s)ds=\frac{1}{e} \qquad (2.9)$$

PROPOSITION 2:

Assume that there is an r-periodic function $r(t)\in C([t_0,\infty),(0,\infty))$. If there exists a $T\geq t_0$

$$\int_{T}^{t}r(s)ds\int_{t}^{\infty}(p(s)-r(s))ds\leq 8e^{2} \qquad (2.10)$$

Then Equation (1.8) has an eventually positive solution and hence non-oscillatory.

THE MAIN RESULTS

In this section, we use a random stationary coordinate change to obtain a conjugation relation between the flow of oscillation in solution of the stochastic optimal delay differential equation and the solution of the random non-autonomous delay differential equation. This method was first proposed by Lisei (2001). We introduce the process $\{\wedge(t,.)\}_{t\in\Re}$ which satisfies the properties of Lemma 1 below. For each $t\in\Re,\ take\ t-r$ for each $w\in\Omega,\ v\in\Re^{d}$ and have the processes $\{\wedge(t,.)\}_{t\in\Re}$ and $\{\Gamma(t,.)\}_{t\in\Re}$ given by

$$\wedge(t,v)=Z(t;t,v\ ) \qquad (3.1)$$

and

$$\Gamma(t,v)=u(t;t,v) \qquad (3.2)$$

The following propostion gives the properties of the random coordinate change (3.1):

PROPOSITION 3:

The stationary coordinate change (3.1) has the following properties:

L_1: $\{\wedge(t,.)\}_{t\in\Re}$ is a continuous $C^{K+1,\in}$ semi – martingale $(with\ 0<\in<\delta)$ such that for all $w\in\Omega,\ \Re^{d}\ni v\to\wedge(t,v)\in\Re^{d}\ is\ a\ C^{K+1}$ diffeomorphism of $\Re^{d}$ and $\{\Gamma(t,.)\}_{t\in\Re}$ a continuous $C^{k,\in}$ semi-martingale $(with\ 0<\in<\delta)$.

L_2: For all $t\geq s,\ v\in\Re^{d}\ and\ a.e.\ w\in\Omega,\ \Lambda(t,v,w)=\Lambda(s,v)+\int_{s}^{t}\mu\left(du,\wedge(u,v)+\int_{s}^{t}\Gamma(u,v)du\right)$

L_3: The processes $\{\wedge(t,.)\}_{t\in\Re}$ and $\{\Gamma(t,.)\}_{t\in\Re}$ are perfectly stationary, that is

$$\Lambda(t,v,u)=\Lambda(0,v,\theta(t,w))\ and\ \Gamma(t,v,u)=\Gamma(0,v,\theta(t,w))\ for\ all\ t\in\Re,v\in\Re,w\in\Omega$$

For details on the properties of the random coordinate change, see Lisei (2001). We now let $\{X(t;t,\phi,u)\}_{t\geq0}$ be the solution of the SDOCS (1.3) and let $\{Z(t,.)\}_{t\geq0}$ be the solution of the random vector DDE (2.2). Also consider coordinate change $\{\wedge(t,.)\}_{t\in\Re}$ with properties as in Proposition 3. Then the following conjugation relation holds:

$$X(t;t,\phi,u)=\Lambda(0,.,\theta(t,w))\circ Z(t,.,w)\circ\Lambda^{-1}(0.,w)\quad for\ all\ t\geq0,\ v\in\Re^{d},w\in\Omega \qquad (3.3)$$

The relation (3.3) expresses the solution X of the SDOCE (1.3) as a conjugation of the solution Z of the random DDE (2.2) and the random coordinate change $\{\wedge(t,.)\}_{t\in\Re}$. Hence the zeros of the trajectory of X are made to correspond to the zeros of the sample path of Z by equation (3.3). It is now possible to obtain information directly about the oscillatory behaviour of the solution X of the SDOCE.

As a result of the fluctuation along the paths of stochastic systems it is usually not possible to analyze the oscillatory behaviour of their solution paths. This is now made possible by the use of equation (3.3). Figure 1 below is a sketch of the corresponding zeros of the sample paths.

REMARK:

In the main result (Theorem 1 below), we establish that if $t - r_{ij}$ satisfies the condition of Proposition 1 for $A_{ij} < 0$ under certain assumptions, then the solution of the SDOCE (1.3) is P-almost surely oscillatory. Also using

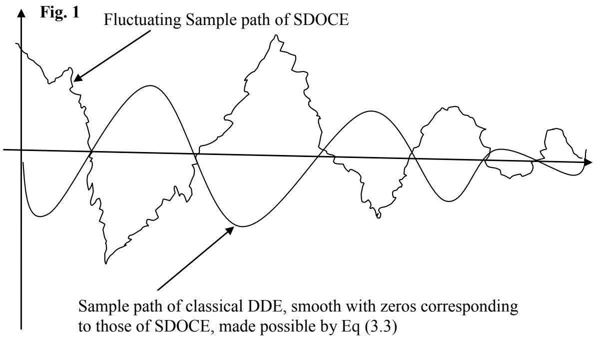

Fig. 1

the condition of proposition 2, we observe that the comparable classical DDE (2.1) can admit a non-oscillatory solution due to the absence of the white noise. We need the following assumptions and Lemma for the main result:

ASSUMPTIONS:

$(i)\ A_{ij}(t),\ r_{ii} > 0,\ i, j = 1,2,3,.........,n$

$(ii)\ r_{ij} \geq 0,\ i, j = 1,2,3,..........,n \ \ with\ r_{ii} \geq r_{ji}$

$(iii)\ \exp\left[\min_{1 \leq i \leq n}\{r_{ii}\}\right]\left[\min_{\substack{1 \leq i \leq n \\ 1 \leq j \leq n}}\left(A_{ii} - \sum_{\substack{j=1 \\ j \neq i}}^{n}|A_{ji}|\right)\right] > 1$

(iv) If $\phi(t) \in \Phi$ and $u(t) \in U$ are some sufficiently small neighbourhood $N(\phi)$ of $\phi(t) \in \Phi$ and $N(\bar{u})$ of $\bar{u}(t) \in U$

Then corresponding to each $\in > o$, there is a $\delta > o$ such that

$$\left\|\lambda(\bar{t},\bar{\phi},\bar{u}) - \lambda(t,\phi,u)\right\| < \varepsilon \ \ for\ each\ \lambda(\bar{t},\overline{\phi,u})\ \ and\ \ all\ \ \bar{t} - \delta < t < \bar{t}\ \ and\ all\ \ \phi \in \Phi, u \in U$$

LEMMA 1:

If $\underset{t \to \infty}{Limit\ Sup} \int_t^{t + r_{ij}} P_{ij}(s)ds > 0,$ for some i, j and x(t) is an eventually positive solution of

$$x^{|}(t) = -\sum_{i=1}^{n} P_{ij}(t)x(t - r_{ij},\ j = 1,2,3,.........,n),\ then\ \ for\ the\ i, j;$$

$$\underset{t \to \infty}{Limit} \inf \frac{x(t - r_{ij})}{x(t)} < \infty$$

THEOREM 1:

Let $A_{ij}, r_{ij}, \phi(t)\ and\ u(t)$ satisfy conditions (i) – (iv) of assumptions (3.1). Then for any initial function $\psi(t) \in C([I_\alpha, t_0], G),$ the stochastic optimal control equation (1.3) has a p-almost sure oscillatory solution on the half interval $[0, \infty).$

Proof:

From the conjugation relation

$$X(t;t,\phi,u) = \Lambda(0,.,\theta(t,w)) \circ Z(t,.,w) \circ \Lambda^{-1}(0.,w)$$

$$for\ all\ t \geq 0,\ u \in \Re^d, w \in \Omega$$

It follows that for the continuous function X to oscillate, the set $Y = \{t \geq 0 : X(t;t,\phi,u) = 0\}$ must satisfy $Sup\, Y = \infty$ by definition which holds only if the set $Y_Z = \{t \geq 0 : Z(t,w) = 0\}$ satisfies $Sup\, Y_Z = 0$ with a positive probability for $t \geq 0\ and\ w \in \Omega.$

Now define $P_{ij}(t,w) = -A_{ij}\Lambda(t - r_{ij})\Lambda^{-1}(t,w)$ which is, a positive continuous function, and hence Z satisfies

$$Z^{|}(t,w) = -\sum_{i=1}^{n} P_{ij}(t,w)Z(t - r_{ij}, w) \qquad (3.4)$$

Assume that there exists a P-almost sure subset $\Omega^* \subset \Omega$ such that

$$\Omega^* = \left\{w \in \Omega : \int_t^{t+\Gamma_n} P_n(s)ds > 0\ and\ \int_{t_0}^{\infty}\left(\sum_{i=1} P_i(t) In\left(e\sum_{i=1}^{n}\int_t^{t+r} P_i(s)ds\right)\right)dt = \infty\right\}$$

We see that as the P_{ij} satisfy the condition of proposition 1 for $w \in \Omega$, it must be that trajectory Z(.,w) is oscillatory and by the relation (3.3), it follows that the $X(t;t,\phi,u;w)$ is oscillatory for $w \in \Omega$. Hence the solution of the SDOCE (1.3) is P-almost surely oscillatory. If not then we employ a technique of assuming the existence of a non-oscillatory solution of (2.2) and then derive a contradiction.

Accordingly, suppose there exists a $t_0 > 0$ and a solution Z(t) of (2.2) such that $Z(t) > 0$

for $t > t_0$ and $Z(t - r_{ij}) > 0$ for $t > t_0 + r_{ij}$, then we have that $Z^{|}(t) < 0$ for $t > t_0 + r_{ij}$. and hence $Z(t) < Z(t - r_{ij})$ for $t > t_0 + 2r_{ij}$.

Define

$$\lambda(t) = \frac{Z(t - r_{ij})}{Z(t)} \text{ for } t > t_0 + 2r_{ij} \text{ and observe that } \lambda(t) > 1$$

. Dividing both sides of (2.2) by Z(t) yields

$$\frac{Z^{|}(t)}{Z(t)} = -\sum_{i=1}^{n} P_{ij}(t)\lambda(t) \text{ for } t > t_0 + 2r_{ij} \qquad (3.5)$$

Integrating both sides of (3.5) from $t - r_{ij}$ to t for $t > t_0 + 3r_{ij}$, one gets

$$In[Z(t)] - In[Z(t - r_{ij})] = -\int_{t-r_{ij}}^{t} P_{ij}(s)ds, \; t > t_0 + 3r_{ij}$$

which implies

$$Log[\lambda(t)] = \int_{t-r_{ij}}^{t} P(s)\lambda(s)ds, \; t < t_0 + 3r_{ij} \qquad (3.6)$$

Define

$$m = \underset{t \to \infty}{Limit} \inf \lambda(t) \qquad (3.7)$$

We observe that $m \geq 1$ which results in two alternatives (i) m is finite (ii) m may be infinite. If (i) and (ii) lead to contradiction, then we are done. Let us take the first case and assume that m is finite, then there exists a sequence $\{t_n\} \to \infty$ as $n \to \infty$ such that $\underset{t \to \infty}{Limit}\, \lambda(t_n) = m$. Equation (3.6) implies

$$Log[\lambda(t_n)] = \int_{t_n - r_{ij}}^{t_n} P_{ij}(s)\lambda(s)ds = \lambda(\xi_n)\int_{t_n - r_{ij}}^{t_n} P_{ij}(s)ds \qquad (3.8)$$

$$\text{where } t_n - r_{ij} < \xi_n < t_n, \; n = 1,2,3,....$$

We now take limits of both sides of equation (3.8) as $n \to \infty$ and obtain

$$Sup\, \lambda \geq 1 \frac{Log[\lambda]}{\lambda} = \frac{1}{e}$$

Using the fact that together with (3.9) results

$$\underset{t \to \infty}{Limit} \inf \int_{t-r_{ij}}^{t} P_{ij}(s)ds \leq \frac{1}{e}$$

which contradicts the first part of the condition of Proposition 1

Now assume case (ii), that is λ is infinite, $(\lambda = \infty)$ we have

$$\underset{t \to \infty}{Limit} \inf \frac{Z(t - r_{ij})}{Z(t)} = \infty \qquad (3.10)$$

Integrating both sides of (3.5) from $t - \frac{r_{ij}}{2}$ to t for $t > t_0 + 3r_{ij}$ yields

$$Z(t) - Z\left(t - \frac{r_{ij}}{2}\right) = -\int_{t-\frac{r_{ij}}{2}}^{t} P_{ij}(s)Z(s - r_{ij}) \qquad (3.11)$$

From (3.11), we have

$$Z(t) - Z\left(t - \frac{r_{ij}}{2}\right) + Z(t - r_{ij})\int_{t-\frac{r_{ij}}{2}}^{t} P_{ij}(s)ds \leq 0 \qquad (3.12)$$

Since $Z(s - r_{ij}) > Z(t - r_{ij})$ for $t - \frac{r_{ij}}{2} \leq s \leq t$.

Dividing both sides of (3.12) by Z(t) and using (3.10) and $Z^{|}(t) = -\sum_{i=1}^{n} P_{ij}(t)Z_j(t - r_{ij})$, we have

$$\underset{t \to \infty}{Limit} \frac{Z\left(t - \frac{r_{ij}}{2}\right)}{Z(t)} = \infty \qquad (3.13)$$

However, by Lemma 1,

$$\underset{t \to \infty}{Limit} \frac{Z\left(t - \frac{r_{ij}}{2}\right)}{Z(t)} < \infty$$

which contradicts (3.13). Also by dividing both sides of (3.11) by $Z\left(t - \frac{r_{ij}}{2}\right)$, we set

$$\frac{Z(t)}{Z\left(t - \frac{r_{ij}}{2}\right)} - 1 + \frac{Z(t - r_{ij})}{Z\left(t - \frac{r_{ij}}{2}\right)} \int_{t-\frac{r_{ij}}{2}}^{t} P_{ij}(s)ds \leq 0$$

which in view of (3.13) and the condition of proposition 1 is again a contradiction, that is, $m = \infty$ is impossible. Hence Z(t) is oscillatory and by (3.3), we have that the solution X of the SDOCE (1.3) is P-almost surely oscillatory.

We remark that the sufficiently large enough increments in the standard Brownian motion help to sustain oscillation irrespective of the magnitude of the time lags. Oscillation now, is the interplay between the time lags and the noise. On the other hand, the comparable classical DDE (2.1), where the noise is absent can have a non-oscillatory solution. This can never happen in the presence of the multiplicative noise.

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