CONTINUOUS DEPENDENCE OF FIXED POINTS OF SOME PARTICULAR MAPS IN COMPLETE METRIC SPACE

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In this paper, we prove the continuous dependence of fixed points in a complete metric space. We show that for function satisfying certain condition with iterative process is continuous and depend on parameter of the space. Results of the investigation revealed that Mann and Kranoselskij iteratives are satisfied with general contractive conditions.

Key words: Continuous dependence, complete metric space, contractive condition and comparison function.

INTRODUCTION

Banach (1922) established a remarkable fixed point theorem known as “Banach contraction principle” which is one of the major result considered in metric fixed point theory. Several results have been published in fixed point theory and iterative approximation procedure for self and non-self-contractive type operators in metric spaces. For a strict contractive type operator, Picard iteration defined by $x_{n+1} = Tx_n$ $(\{x_n\}_{n=0}^{\infty}\ n = 0, 1, 2, \ldots)$ has been considered in approximating the unique fixed point when the contractive conditions are slightly weaker, then the Picard iteration need not converge to a fixed point of the operator T and some other iterative procedures will be considered (Berinde, 2007). This work used contractive types of Mann, Krasnoselskij and Ishikawa schemes.

Let (X, d) be a metric space and $T: XxX \to X$ a selfmap of X with fixed point $p \in F_T$. For a given $x_0 \in X$, we consider the sequence of iteration $\{x_n\}_{n=0}^{\infty}$ determined by the successive iteration method:

$$\begin{cases} x_n = T(x_{n-1}) = T^n(x_0) & n = 1,2,\ldots, \\ x_{n+1} = T(x_n), & n = 0,1,2,\ldots, \end{cases} \quad (1.1)$$

Picard iterative process (1.1) has been used to approximate the fixed points of mappings satisfying the relation:

$$d(Tx,Ty) \leq ad(x,y),\ for\ all\ x,y \in X\ and\ a \in [0,1) \quad (1.2)$$

Inequality (1.2) is referred to as contraction principle.

Among the iterative procedures that generalize (1.1) are:

(i) for $x_0 \in X$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = (1-\beta)x_n + \beta Tx_n \qquad n = 0,1,\ldots, \quad (1.3)$$

$where\ \{\beta\} \subset [0,1]$ satisfying certain appropriate conditions is called a Kranoselskij iteration, Berinde (2007).

(ii) for $x_0 \in X$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n Tx_n, \quad n = 0,1,2,\ldots, \quad (1.4)$$

Where, $\{\alpha_n\}_{n=0}^{\infty} \subset [0,1]$ satisfying some conditions, is called the Mann iteration.

(iii) for $x_0 \in X$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_n = (1-\alpha_n)x_n + \alpha_n T[(1-b_n)x_n + b_n Tx_n, \qquad n = 0,1,\ldots, \quad (1.5)$$

Where, $\{\alpha_n\}_{n=0}^{\infty}, \{b_n\}_{n=0}^{\infty} \subset [0,1]$ satisfying certain appropriate condition, is called Ishikawa iteration scheme.

Equation (1.5) in a system form is:

$$\begin{cases} y_n = (1-b_n)x_n + b_n T x_n \\ x_{n+1} = (1-a_n)x_n + a_n T y_n \end{cases} \qquad n = 0,1,\dots,$$

Ishikawa iteration is regarded as double Mann iteration with two different parameter sequences (2).

(iv) Let X be a Banach space, and suppose T is a mapping of X to X, the Kirk's iteration procedure is given by $x_0 \in X$ and

$$x_{n+1} = \alpha_0 x_0 + \alpha_1 T^1 x_1 + \alpha_2 T^2 x_2 + \cdots + \alpha_k T^k x_k \qquad (1.6)$$

Where, k is a fixed integer, $k \geq 0, \alpha_i \geq 0, for\ i = 0,1,\dots,k,$ $\alpha_1 \geq 0$ and $\alpha_0 + \alpha_1 + \cdots + \alpha_k = 1$.

Recently, Rauf et al. (2017) introduced some new implicit Kirk-type iterative schemes to generalize convex metric spaces in order to approximate fixed points for general class of quasi-contractive operators. The strong convergence, T-stability, equivalency, data dependence and convergence rate of these results were explored. The results are faster and better, in term of speed of convergence, than the corresponding results of Olatinwo (2009), Chugh and Kumar (2012), Hussain et al. (2012), Gursoy et al. (2013) and Akewe et al. (2014). These results also improved and generalized several existing iterative schemes in the literature and they provided analogues of the corresponding results of other spaces, namely: normed spaces, CAT(0) spaces and so on.

Also, the following researchers worked on fixed point theorem and its applications: Debnath (2014), Eshi et al. (2016), Choudhury et al. (2016), Debnath et al. (2014) and Neog and Debnath (2017).

Some results concerning the continuous dependence of the fixed points in a complete metric space are established in this paper by using some general contractive conditions. The result established for the fixed point is similar to those of Olatinwo (2010) by employing a weaker contractive type.

Basic definitions and preliminaries

The needed definitions and lemmas are stated as follows.

Let $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ be a function in connection with the function φ, then the following properties are valid:

i. φ is monotone increasing;
ii $\varphi(t) < t$ for all $t > 0$;
iii $\varphi(0) = 0$;
iv $\{\varphi^n(t)\}$ converges for all $t \geq 0$;
v $\sum_{n=0}^{\infty} \varphi^n(t)$ converges for all $t > 0$;
vi $t - \varphi(t) \to \infty$ as $t \to \infty$; and
vii φ is subadditive.

Definition 2.1

A function $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ is called a comparison function if it satisfies the following conditions:
i φ is monotone increasing; and
ii $\lim_{n\to} \varphi^n(t) = 0$ for all $t \geq 0$.

Definition 2.2

A comparison function satisfying $t - \psi(t) \to \infty\ as\ t \to \infty$ is called a strict comparison function.

Definition 2.3

A function satisfying the following condition is called a c – comparison function
(a) φ is monotone increasing; and
(b) $\sum_{n=0}^{\infty} \varphi^n(t)$ converges for all $t > 0$.

Further examples and definitions of comparison function are given (Imoru et al., 2006; Kazimierz and Williams, 2000, 1990; Mann, 1953; Mohamed and Williams, 2001; Olatinwo, 2008; Ravi et al. 2000; William and Brailey, 2001; Zeidler, 1986).

Remarks 2.1

Every comparison function satisfies $\varphi(0) = 0$.

In (2007), Berinde formulated the continuous dependence of the fixed points on a parameter λ in the following general context:
Let (X, d) be a metric space, (Y, τ) a topological space and $T : X$ x $Y \to X$ a family of operators depending on the parameter $\lambda \in Y$, where Y is a parameter space. Assume that $T_\lambda = (.,\lambda)$, $\lambda \in Y$

and consider the operator $U : Y \to X$, then $U(\lambda) = x_\lambda^*$ for all $\lambda \in Y$. The objectives of this study are to obtain sufficient conditions on T that guarantee the continuity of U.

In this paper, the following contractive conditions for a continuous mapping $T_\lambda : X$ x $Y \to X$ shall be employed:

a. A strict comparison function $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ such that for all $x, y \in X$

$$d(T(x,\lambda), T(y,\lambda)) \leq \varphi(d(x,y)) \quad 2.1$$

b. A real number $a \geq 0$, a monotone increasing function $\theta : \mathbb{R}_+ \to \mathbb{R}_+$ with $\theta(0) = 0$ and a strict comparison $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ such that

$$d(T(x,\lambda), T(y,\lambda)) \leq ad(x,Tx) + \theta d(y,Ty) + \varphi d(x,y), \quad \text{for all } x, y \in X \quad 2.2$$

c. A real number $L \geq 0$ and a strict comparison function $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ such that for all $x, y \in X$

$$d(T(x,\lambda), T(y,\lambda)) \leq Ld(x,Tx) + \varphi d(x,y) \quad 2.3$$

In this section, we prove the major contribution in this paper. To this end, the continuous dependent results are shown to be valid with Mann and Krasnoselskij iterative processes. The following theorems are considered:

Theorem 3.1

Let (X,d) be a complete metric space and (Y,τ) a topological space. If $T : X$ x Y $\to X$ is a continuous mappings for which there exists a strict comparison function $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ such that (2.2) is satisfied for all $x, y \in X$ and $\lambda \in Y$ where $T_\lambda x = T(x,\lambda)$. Let x_λ^* be the unique fixed point of T_λ. Suppose $\{x_n\}_{n=0}^\infty$ is the Mann iterative process defined by (1.4) with $\{\alpha_n\}_{n=0}^\infty \subset [0,1]$, then, the mapping $U : Y \to X$ given by $U(\lambda) = x_\lambda^*$, $\lambda \in Y$ is continuous.

Proof. Let $\lambda_1, \lambda_2 \in Y$. Then, we shall apply the general contractive conditions in the proof of the theorem.

RESULTS

$$d(x_{\lambda1}^*, x_{\lambda2}^*) = (1-\alpha_{\lambda1})d(x_{\lambda1}^*, x_{\lambda2}^*) + \alpha_{\lambda1} d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda2}^*) + (\alpha_{\lambda2} - \alpha_{\lambda1})x_{\lambda2}^* + (\alpha_{\lambda1} - \alpha_{\lambda2})T_{\lambda2}x_{\lambda2}^*)$$

$$\leq (1-\alpha_{\lambda1})d(x_{\lambda1}^*, x_{\lambda2}^*) + \alpha_{\lambda1}[d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda1}^*) + d(T_{\lambda2}x_{\lambda1}^*, T_{\lambda2}x_{\lambda2}^*)] + (\alpha_{\lambda2} - \alpha_{\lambda1})x_{\lambda2}^* + (\alpha_{\lambda1} - \alpha_{\lambda2})T_{\lambda2}x_{\lambda2}^*)$$

$$= (1-\alpha_{\lambda1})d(x_{\lambda1}^*, x_{\lambda2}^*) + \alpha_{\lambda1} d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda1}^*) + \alpha_{\lambda1} d(T_{\lambda2}x_{\lambda1}^*, T_{\lambda2}x_{\lambda2}^*) + (\alpha_{\lambda2} - \alpha_{\lambda1})x_{\lambda2}^* + (\alpha_{\lambda1} - \alpha_{\lambda2})T_{\lambda2}x_{\lambda2}^*)$$

$$\leq (1-\alpha_{\lambda1})d(x_{\lambda1}^*, x_{\lambda2}^*) + \alpha_{\lambda1} d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda1}^*) + \alpha_{\lambda1}[\varphi d(x_{\lambda1}^*, x_{\lambda2}^*)] + (\alpha_{\lambda2} - \alpha_{\lambda1})x_{\lambda2}^* + (\alpha_{\lambda1} - \alpha_{\lambda2})T_{\lambda2}x_{\lambda2}^*)$$

$$d(x_{\lambda1}^*, x_{\lambda2}^*) - \alpha_{\lambda1}\varphi d(x_{\lambda1}^*, x_{\lambda2}^*) \leq \alpha_{\lambda1} d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda1}^*) + (1-\alpha_{\lambda1})d(x_{\lambda1}^*, x_{\lambda2}^*) + (\alpha_{\lambda2} - \alpha_{\lambda1})x_{\lambda2}^* + (\alpha_{\lambda1} - \alpha_{\lambda2})T_{\lambda2}x_{\lambda2}^*)$$

Hence,

$$(1-\alpha_{\lambda1}\varphi)d(x_{\lambda1}^*, x_{\lambda2}^*) \leq \alpha_{\lambda1} d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda1}^*) + (1-\alpha_{\lambda1})d(x_{\lambda1}^*, x_{\lambda2}^*) + (\alpha_{\lambda2} - \alpha_{\lambda1})x_{\lambda2}^* + (\alpha_{\lambda1} - \alpha_{\lambda2})T_{\lambda2}x_{\lambda2}^*)$$

Since T is continuous and φ is a strict comparison function for $\lambda_2 \to \lambda_1$, then,

$$\alpha_{\lambda1} d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda1}^*) \to 0 \quad as\ \lambda_2 \to \lambda_1$$

and

$$(1-\alpha_{\lambda1})d(x_{\lambda1}^*, x_{\lambda2}^*) \to 0 \quad as\ \lambda_2 \to \lambda_1$$

then

$$(\alpha_{\lambda2} - \alpha_{\lambda1})x_{\lambda2}^* \to 0 \quad as\ \lambda_2 \to \lambda_1$$

And $(\alpha_{\lambda1} - \alpha_{\lambda2})T_{\lambda2}x_{\lambda2}^*) \to 0 \quad as\ \lambda_2 \to \lambda_1$

which implies

$$d(x_{\lambda1}^*, x_{\lambda2}^*) \to 0 \quad as\ \lambda_2 \to \lambda_1$$

Therefore,

$$d(U(\lambda1), U(\lambda2)) \to 0 \quad as\ \lambda2 \to \lambda1$$

Hence, the mapping $U : Y \to E$, defined by $U(\lambda) = x_\lambda^*$, $\lambda \in Y$ is continuous and depends on λ.■

Theorem 3.2

Let (X,d) be a complete metric space and (Y,τ) a topological space. If T: X x $Y \to X$ is a continuous mapping for which $\theta : \mathbb{R}_+ \to \mathbb{R}_+$ is a monotone increasing function such that $\theta(0) = 0$ and $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ a strict comparison function such that (2.3) is satisfied for all $x, y \in X$ and $\lambda \in Y$ where $T_\lambda x = T(x,\lambda)$. Let x_λ^* be the unique fixed point of T_λ. Suppose $\{x_n\}_{n=0}^{\infty}$ is the Kranoselskij iterative process defined by (1.3) with $\beta \geq 0$. Then the mapping $U : Y \to X$, given by $U(\lambda) = x_\lambda^*,\ \ \lambda \in Y$ is continuous.

Proof. Let $\lambda_1, \lambda_2 \in Y$. Then,

$$d(x_{\lambda1}^*, x_{\lambda2}^*) = (1-\beta)d(x_{\lambda1}^*, x_{\lambda2}^*) + \beta d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda2}^*)$$
$$\leq (1-\beta)d(x_{\lambda1}^*, x_{\lambda2}^*) + \beta[d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda1}^*) + d(T_{\lambda2}x_{\lambda1}^*, T_{\lambda2}x_{\lambda2}^*)]$$
$$= (1-\beta)d(x_{\lambda1}^*, x_{\lambda2}^*) + \beta d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda1}^*) + \beta d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda2}^*)$$
$$\leq (1-\beta)d(x_{\lambda1}^*, x_{\lambda2}^*) + \beta d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda1}^*) + \beta[ad(x_{\lambda1}^*, T_{\lambda1}x_{\lambda1}^*) + \theta d(x_{\lambda2}^*, T_{\lambda2}x_{\lambda2}^*) + \varphi d(x_{\lambda1}^*, x_{\lambda2}^*)]$$
$$\leq (1-\beta)d(x_{\lambda1}^*, x_{\lambda2}^*) + \beta d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda1}^*) + \beta\varphi d(x_{\lambda1}^*, x_{\lambda2}^*)$$
$$d(x_{\lambda1}^*, x_{\lambda2}^*) - \beta\varphi d(x_{\lambda1}^*, x_{\lambda2}^*) \leq \beta d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda1}^*) + (1-\beta)d(x_{\lambda1}^*, x_{\lambda2}^*)$$

Hence

$$(1-\beta\varphi)d(x_{\lambda1}^*, x_{\lambda2}^*) \leq \beta d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda1}^*) + (1-\beta)d(x_{\lambda1}^*, x_{\lambda2}^*)$$

Since T is continuous and φ is a strict comparison function, for $\lambda_2 \to \lambda_1$. Then,

$$\beta d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda1}^*) \to 0 \quad as\ \lambda_2 \to \lambda_1$$

and

$$(1-\beta)d(x_{\lambda1}^*, x_{\lambda2}^*) \to 0 \quad as\ \lambda_2 \to \lambda_1$$

then

$$d(x_{\lambda1}^*, x_{\lambda2}^*) \to 0 \quad as\ \lambda_2 \to \lambda_1$$

Therefore

$$d\big(U(\lambda1), U(\lambda2)\big) \to 0 \quad as\ \lambda_2 \to \lambda_1$$

Hence, the mapping $U: Y \to E$, defined by $U(\lambda) = x_\lambda^*,\ \ \lambda \in Y$ is continuous and depends on λ.

Theorem 3.3

Let (X,d) be a complete metric space and (Y,τ) a topological space. If $T : X$ x Y $\to X$ is a continuous mapping satisfying (2.4). Suppose that $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ is a strict comparison function. Let x_λ^* be the unique fixed point of T_λ, where $T_\lambda x = T(x,\lambda)$, for all $x, y \in X$ and $\lambda \in Y$. Suppose $\{x_n\}_{n=0}^{\infty}$ is the Mann iterative process defined by (1.4) with $\{\alpha_n\}_{n=0}^{\infty} \subset [0,1]$. Then the mapping $U : Y \to X$, given by $U(\lambda) = x_\lambda^*,\ \ \lambda \in Y$ is continuous.

Proof. Let $\lambda_1, \lambda_2 \in Y$. Then,

$$d(x_{\lambda1}^*, x_{\lambda2}^*) = (1-\alpha_{\lambda1})d(x_{\lambda1}^*, x_{\lambda2}^*) + \alpha_{\lambda1}d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda2}^*) + (\alpha_{\lambda2} - \alpha_{\lambda1})x_{\lambda2}^* + (\alpha_{\lambda1} - \alpha_{\lambda2})T_{\lambda2}x_{\lambda2}^*)$$
$$\leq (1-\alpha_{\lambda1})d(x_{\lambda1}^*, x_{\lambda2}^*) + \alpha_{\lambda1}[d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda1}^*) + d(T_{\lambda2}x_{\lambda1}^*, T_{\lambda2}x_{\lambda2}^*)] + (\alpha_{\lambda2} - \alpha_{\lambda1})x_{\lambda2}^* + (\alpha_{\lambda1} - \alpha_{\lambda2})T_{\lambda2}x_{\lambda2}^*)$$
$$= (1-\alpha_{\lambda1})d(x_{\lambda1}^*, x_{\lambda2}^*) + \alpha_{\lambda1}d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda1}^*) + \alpha_{\lambda1}d(T_{\lambda2}x_{\lambda1}^*, T_{\lambda2}x_{\lambda2}^*) + (\alpha_{\lambda2} - \alpha_{\lambda1})x_{\lambda2}^* + (\alpha_{\lambda1} - \alpha_{\lambda2})T_{\lambda2}x_{\lambda2}^*)$$
$$\leq (1-\alpha_{\lambda1})d(x_{\lambda1}^*, x_{\lambda2}^*) + \alpha_{\lambda1}d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda1}^*) + \alpha_{\lambda1}[Ld(x_{\lambda1}^*, T_{\lambda1}x_{\lambda1}^*) + \varphi d(x_{\lambda1}^*, x_{\lambda2}^*)] + (\alpha_{\lambda2} - \alpha_{\lambda1})x_{\lambda2}^* + (\alpha_{\lambda1} - \alpha_{\lambda2})T_{\lambda2}x_{\lambda2}^*)$$
$$\leq (1-\alpha_{\lambda1})d(x_{\lambda1}^*, x_{\lambda2}^*) + \alpha_{\lambda1}d(T_{\lambda1}x_{\lambda1}^*, T_{\lambda2}x_{\lambda1}^*) + \alpha_{\lambda1}\varphi d(x_{\lambda1}^*, x_{\lambda2}^*) + (\alpha_{\lambda2} - \alpha_{\lambda1})x_{\lambda2}^* + (\alpha_{\lambda1} - \alpha_{\lambda2})T_{\lambda2}x_{\lambda2}^*)$$

then
$d(x^*_{\lambda 1}, x^*_{\lambda 2}) - \alpha_{\lambda 1}\varphi d(x^*_{\lambda 1}, x^*_{\lambda 2}) \le \alpha_{\lambda 1} d(T_{\lambda 1}x^*_{\lambda 1}, T_{\lambda 2}x^*_{\lambda 1}) + (1-\alpha_{\lambda 1})d(x^*_{\lambda 1}, x^*_{\lambda 2}) + (\alpha_{\lambda 2} - \alpha_{\lambda 1})x^*_{\lambda 2} + (\alpha_{\lambda 1} - \alpha_{\lambda 2})T_{\lambda 2}x^*_{\lambda 2})$

Hence
$$(1-\alpha_{\lambda 1}\varphi)d(x^*_{\lambda 1}, x^*_{\lambda 2}) \le \alpha_{\lambda 1} d(T_{\lambda 1}x^*_{\lambda 1}, T_{\lambda 2}x^*_{\lambda 1}) + (1-\alpha_{\lambda 1})d(x^*_{\lambda 1}, x^*_{\lambda 2}) + (\alpha_{\lambda 2} - \alpha_{\lambda 1})x^*_{\lambda 2} + (\alpha_{\lambda 1} - \alpha_{\lambda 2})T_{\lambda 2}x^*_{\lambda 2})$$

Since T is continuous and φ is a strict comparison function, for $\lambda_2 \to \lambda_1$. Then,
$\alpha_{\lambda 1} d(T_{\lambda 1}x^*_{\lambda 1}, T_{\lambda 2}x^*_{\lambda 1}) \to 0 \;\; as\; \lambda_2 \to \lambda_1$
and
$(1-\alpha_{\lambda 1})d(x^*_{\lambda 1}, x^*_{\lambda 2}) \to 0 \;\; as\; \lambda_2 \to \lambda_1$
then
$$(\alpha_{\lambda 2} - \alpha_{\lambda 1})x^*_{\lambda 2} \to 0 \;\; as\; \lambda_2 \to \lambda_1$$
and
$(\alpha_{\lambda 1} - \alpha_{\lambda 2})T_{\lambda 2}x^*_{\lambda 2}) \to 0 \;\; as\; \lambda_2 \to \lambda_1$
hence
$d(x^*_{\lambda 1}, x^*_{\lambda 2}) \to 0 \;\; as\; \lambda_2 \to \lambda_1$
Therefore
$d\big(U(\lambda 1), U(\lambda 2)\big) \to 0 \quad as\; \lambda 2 \to \lambda 1$
Hence the mapping $U: Y \to E$, defined by $U(\lambda) = x^*_{\lambda}, \;\; \lambda \in Y$ is continuous and depends on λ.

CONCLUSION

The study has proved some results on continuous mappings for which there exists a strict comparison and monotone increasing functions by using Mann and Kranoselskij processes. Furthermore, continuous dependence of fixed points in complete metric space were established. The results are extension of continuous dependence of fixed point to some known processes.

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