

A GENERALIZED SEMI-EMPIRICAL T_c EXPRESSION OF THE ISO-SUPERCONDUCTIVITY THEORY

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ABSTRACT

As the international scientific community celebrates the first century of the discovery of superconductivity this year, 2011, an outstanding problem is achieving a generalized mechanism of the Cooper pair formation (CPF). In this current study, it has been shown that at short distances, the CPF is by a nonlinear, nonlocal and nonhamiltonian strong hadronic-type interactions due to deep wave-overlapping of spinning particles leading to Hulthen potential that is attractive between two electrons in singlet couplings while at large distance the CPF is by superexchange interaction which is purely a quantum mechanical affairs. It is observed that for both distances the control parameter responsible for the superconducting state is the effective valence z . Therefore a semi-empirical T_c expression depending on the effective valence is obtained. This T_c expression gives the same results as the successful BCS T_c expression but unlike the latter whose predicting power is limited to 25 K, the former predicts higher temperatures even beyond room temperature. The implication is that unlike the BCS theory that is restricted to conventional superconductors, the iso-superconductivity theory can be used to account for the conventional and the non-conventional su-

INTRODUCTION

The international scientific community has declared this year, 2011, as the centenary celebration of the discovery of superconductivity (SC) in Hg at about 4 K by the Dutch physicist, Heike kamerlingh Onnes on August 8, 1911. The defining electromagnetic property of superconductivity is the complete disappearance of the dc electrical resistivity at and below a certain temperature called the critical or transition temperature T_c . In other words, if a superconducting material with $T \leq T_c$ is an element in a dc network, the voltage drop across it will be zero. Furthermore, if a superconductor is in the form of a loop and a current is induced in it, this current will persist without measurable decay (Rickayzen, 1969). Today, there are metallic/non-metallic, conventional/nonconventional, doped/undoped and low/high T_c superconducting materials (Hott *et al.*, 2005; Akpojotor and Animalu, 2011). The critical temperature which is material specific (Animalu, 1977) can be obtained from experiment or calculated from the theoretical expression derived from the theory being used to explain the physics of superconductivity. The theoretical T_c is usually theory specific, that is, it varies with theory. The reason why T_c varies with theory is obvious; the theories vary in their correct explanation of the physics of superconductivity.

There have been quite a number of failed theories postulated since 1911 to account for superconductivity and these include those of some of the respected 20th century physics intellectuals such as Einstein, Heisenberg, Feynman, etc (Schmalian, 2011).

The basic requirement of any theory of superconductivity is to first provide a mechanism of the Cooper pair formation (CPF) and its coherent propagation. Here a Cooper pair is the bound state of two electrons which is contrary to the prediction of quantum mechanics. Therefore many of the theories that have been proposed based on this quantum mechanical limitation seeks for approximate means to gluon the electrons to form the Cooper pair. For example, Leon Neil Cooper was able to show in 1956 that under a favourable condition, the electrons are gluoned by the mediation of the phonon to form the Cooper pair (Cooper, 1956) (see Fig. 1a). This is the so called electron-phonon interaction (EPI) mechanism which Bardeen-Cooper-Schrieffer (BCS) adopted in 1957 to achieve their theory of superconductivity using a variational approach: the salient feature of this theory is that the EPI will lead to the formation of an ensemble of Cooper pairs which can propagate coherently with negligible resistance and this is the superconducting state of the parent ma-

terial.

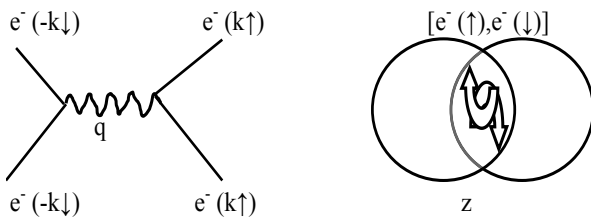


Fig. 1: Attractive interaction of the electron pair (a) due to virtual phonon exchange in the BCS model and (b) due to overlapping electron wavefunctions in the superexchange interaction.

The BCS theory has been used to account for a number of metallic and intermetallic superconductivity and all such materials are known as conventional superconductors and their transition temperatures are low (Bennemann and Ketterson, 2008). This BCS theory has led to a number of important applications like superconducting magnets for laboratory use (spanning from small-scale laboratory experiments to the LHC's bending magnets), and importantly, for MRI systems as well as explanation of puzzling experimental data such as nuclear magnetic resonance (NMR) relaxation rate and Josephson tunneling (Buckel and Kleiner, 2004). However, the BCS failed to explain the Meissner effect which is a fundamental property of superconductors and has been proven incapable of predicting high – temperature superconductors and providing the guidelines to search for new materials. Therefore the BCS theory did not only fail to predict the relatively high T_c of the superconducting copper oxide compounds commonly known as the cuprates discovered by Bednorz and Mueller (1986) but also failed to account for this class of superconductors (Heid *et al.*, 2008). In general, the BCS has failed in its application to a number of new classes of superconductors discovered since 1970 (Hirsch, 2009; Akpojotor and Animalu, 2011). To account for these superconductors now collectively known as non-conventional superconducting materials, there has been a deluge of proposals of new theories which generalized the BCS theory by replacing the phonon with other bosons (Schrieffer, 1989; Pashkin *et al.*, 2010), introduce an interplay of the EPI and other mechanisms (Heid *et al.*, 2008.) or are formulated from non – EPI mechanisms (see Akpojotor and Animalu

(2011) and references therein) as well as those that even question the validity of the BCS theory (Hirsch, 2009). However, the partial successes of these theories for the superconducting materials they are developed for, means the main properties of the superconducting phenomena are still poorly understood and new concepts are needed (Gandzha. and Kadeisvily, 2010, Stockert *et al.*, 2011).

At its meeting in College Park on October 16, 1999, the Executive Board of the American Association of Physics Teachers asserted that: “No scientific theory, no matter how strongly supported by available evidence is final and unchangeable; any good theory is always exposed to the possibility of being overthrown by new observational evidence. That is at the very heart of the process of science.” This is the sequence of human progress: the solution to a problem at any given time is usually limited to the available knowledge and therefore new solutions will emerged with new and enhanced knowledge. For as Dirac (1989) said, this “frequently happens with mathematical procedure in research; the solving of one difficulty leads to another. You may think that no real progress is then made, but this is not so, because the second difficulty is more remote than the first. It may be that the second difficulty was really there all the time and was only brought into prominence by the removal of the first one.” On the contrary, those studies which may not be the correct, have their own unique way of contributing to progress in the research field because if we mimic the inventive of Thomas Edison, “We are exposed to several ways of not getting the right thing.”

The purport of all these is that, in general, apart from some well-understood aspects of superconductivity, there are still many areas in which people are ‘believing’ several suggestions. The word ‘belief’ is therefore prevalent: this word is usually used to express those ideas about which we have no definite knowledge. In other words, the word is used to describe a psychological disposition in which individuals find themselves when they have no precise knowledge of a thing. The importance of ‘belief’ is that it is the next step to knowing.

It is therefore necessary that as we

celebrate its centenary discovery, there should be a concerted search for a generalized theory of superconductivity. It is expected that such a theory should be able to account for the properties of the superconductors with sharp fairness. For example, it should be able to fairly predict the T_c , that is, even if the theory cannot predict the T_c accurately because of no availability of precise input parameters for now, it should give the correct order of magnitude. In other words, one of the practical tests of a good theory is that the T_c calculated from it should agree or at least, have the same order of magnitude with experiment. This is the approach I have adopted in this current study. Basically, our recent proposal of achieving the Cooper pair formation from the appropriate bonding of the elements (Akpojotor and Animalu, 2011) will be used to formulate a general semi-empirical critical temperature of superconducting materials. Thus this paper will be organized as follows. In the next section, the formation of the Cooper pair from the attractive Hulthen potential will be developed. This will then be used to obtain the semi-empirical critical temperature. The possible application of this critical temperature will then be discussed. This will be followed by a summary and a conclusion.

THE HADRONIC COOPER PAIR FORMATION

The possible means of naturally achieving the CPF from the appropriate bonding of the elements emanated from the Santilli's proposal in 1978 (Santilli, 1978) to build the foundation of hadronic mechanics wherein a bound state of one electron and one positron at a short distance ($< 1 \text{ F} \sim 10^{-13} \text{ cm}$) with nonlocal, nonlinear and nonpotential is due to deep overlapping of their wavepackets. Animalu observed that at such distances, the magnetically induced Hulthen potential which is an attractive force will dominate the Coulomb repulsion between two electrons to allow them to bond into singlet coupling as in the CPF in the cuprates (Animalu, 1991; 1994; Animalu and Santilli, 1995). This has been generalized as the Santilli-Shillady-Animalu strong valence which states that a nonlinear, nonlocal and nonhamiltonian interactions due to wave-overlapping of spinning

particles at short distances are always attractive when in singlet couplings and such to absorbs repulsive or attractive Coulomb interactions, resulting in total strongly attractive interactions irrespective of whether the Coulomb interaction are attractive or repulsive (Gandzha. and Kadeisvily, 2010).

Classically, the motion of a body can be described by its dynamic variables such its velocity, mass and time. However, the Heisenberg principle restrict such description in quantum mechanics to variables such as momentum, position and time (not often) i.e. $\psi(p,r,t)$. This restriction makes it necessary to re-express the total energy in terms of p , r , and t only. This can be depicted mathematically by the conventional quantum mechanical equation in relative coordinates and reduced mass for two electrons in singlet coupling (Santilli and Shillady, 1999) as

$$(p^2/m + e^2/r) \psi(r) = E \psi(r) \quad (1)$$

where m is the mass of the electron.

It is obvious that the potential in Eq.(1) is the repulsive Coulomb force between the point-like charges of the electrons. Thus the replacement of the classical variables in Eq. (1) by their corresponding quantum mechanical operators will result in a Hamiltonian for two repelling electrons. This is the origin of the limitation of quantum mechanics to account for superconductivity since the repelling electrons are not expected to bind to form the Cooper pairs.

In nature, however, the electrons have extended wavepackets of the order of 1 fm as shown in Fig. (1b). Therefore there will be mutual overlap/penetration of the wavepackets of the two electrons which allows them to have a nonlinear, nonlocal and nonpotential interactions that will result to valence bond of the Cooper type. One possible way to achieve an invariant representation of these interactions is to exit from the class of unitary equivalence of for quantum mechanics,

$$UU^+ = UU^\dagger = I \quad (2)$$

via an isounitary transformation by projecting into a conventional nonunitary form

$$UU^+ \neq I, UU^\dagger = I^* = 1/T. \quad (3)$$

Taking into account Eqs.(1) and (3), one can project out a different eigenvalue E' different from the one E in Eq. (1):

$$\begin{aligned}
& U [(p^2/m + e^2/r) \psi(r)] U^\dagger \\
& = [(Up^2U^\dagger)/m + (e^2/r)UU^\dagger] (UU^\dagger)^{-1} [U\psi(r)U^\dagger] \\
& = [1/m)p^* T p^* T + e^2/r] \psi^*(r) = E' \psi^*(r). \quad (3)
\end{aligned}$$

At this point, Santilli and Shillady (1999) introduce the following realization of the non-unitary transform,

$$\begin{aligned}
& UU^\dagger = I^* = 1/T = \\
& = e^{\int [\psi(r)/\psi^*(r)] \int \psi_1^\dagger(r) \psi_2(r) d^3(r)} = 1 + [\psi(r)/\psi^*(r)] \int \\
& \psi_1^\dagger(r) \psi_2(r) d^3(r) + \dots \quad (4)
\end{aligned}$$

where ψ and ψ^* are the solutions of the unitary and nonunitary equations, and ψ_k , $k = 1, 2$, are the conventional quantum mechanical wavefunctions of the two electrons.

It is evident that, as desired, the above isounit represents interactions that are: nonlinear, because dependent in a nonlinear way in the wavefunctions; nonlocal, because inclusive of a volume integral; and nonpotential, because not representable with a Hamiltonian. Additionally, for all mutual distances between the valence electrons greater than 1 fm, the volume integral of Eq. (4) is null with the crucial limit

$$\lim_{r \text{ bigger } 1 \text{ fm}} I^* = 1, \quad (5)$$

under which the quantum scenario can be identically and uniquely recovered from that of the hadronic.

Santilli and Shillady (1999) solved the above equations in all details. First, by inserting isounit in Eq.(4) into Eq. (3), they obtained the isoequation here projected on a conventional Hilbert space

$$[p^2/2m' + e^2/r - V_0 e^{-br}/(1 - e^{-br})] \psi^*(r) = E' \psi^*(r), \quad (6)$$

where m' represents the isorenormalization of the mass caused by nonpotential interactions, and one recognizes the emergence of the attractive Hulthen potential

$$V_{\text{Hulthen}} = V_0 e^{-br}/(1 - e^{-br}). \quad (7)$$

But the Hulthen potential is known to behave like the Coulomb potential at short distances and be much stronger than the latter. Therefore, Eq. (6) admits the excellent approximation

$$[p^2/2m' - V' e^{-br}/(1 - e^{-br})] \psi^*(r) = E' \psi^*(r), \quad (8)$$

where the new constants V' reflects the "absorption" of the repulsive Coulomb potential by the much stronger attractive Hulthen potential.

Thus Eq.(8) depicts a hadronic mechanical equation in relative coordinates and reduced mass for two electrons in singlet coupling with a strongly attractive force capable of forming the Cooper pair, as requested by experimental evidence. This is the foundation of the isosuperconductivity theory. It is pertinent to point out that at large distances ($> 1 \text{ fm}$), the Hulthen potential no longer dominates and it has been suggested by Akpojotor (2008) that the CPF is by superexchange interaction which naturally affects electrons that are close enough to have (no deep) overlapping wavefunctionis and this is purely a quantum mechanical affairs (Gandzha. and Kadeisvily, 2010). It follows then that the results to be obtained taking into account the CPF at short distances can also be obtained as the approximation of the hadronic-type from the CPF by superexchange interaction at large distances.

THE SEMI-EMPERICAL CRITICAL TEMPERATURE AND ITS APPLICATION

The two early independent studies of Maxwell (1950) and Reynolds *et al.* (1950) showed that that the T_c of superconductors depends on the isotopic mass of the lattice. This gave birth to the property of isotopic effect in superconductivity in which two isotopes of the same superconducting materials have different critical temperatures because the T_c is proportional to the isotopic mass, that is,

$$T_c \propto M^{-\beta} \quad (9)$$

where M is the isotopic mass and b is the isotopic shift.

In the BCS theory, it is now a textbook knowledge that the isotopic effect emanates from the differences in the bandwidths or relevant/characteristic frequency of the mediating phonons w such that

$$\omega = (k/M)^{1/2} \quad (10)$$

where k is a constant.

It is easy to observe in Eqs (9) and (10) that as M is decreased, both the T_c and w will increase. This is embedded in the BCS T_c expression which has been successful for the conventional superconductors,

$$T_c \approx \frac{\hbar\omega}{k_\beta} e^{-1/g}, \tag{11}$$

where \hbar is the Plank's constant and k_β is the Boltzman's constant while the coupling factor g is defined by

$$g = \lambda + \mu \approx N(E_F)V_{EPI} \tag{12}$$

where λ is the EPI coupling constant, $N(E_F)$ is the density of state at the Fermi level and V_{EPI} is the EPI coupling strength.

Observe that the effect of the Coulomb repulsive parameter μ^* is considered negligible, that is, the V_{EPI} is completely dominant. This is a mathematical assumption introduced for convenience: that under an ambiguous favorable conditions, the V_{EPI} should dominate μ^* . However, it is argued here that the V has been simulating the effect of the strong hadronic valence, that is, when the electrons wavepackets overlap deeply, the interaction is dominated by the Hulthen potential. Interestingly, it has been shown in previous works (Animalu, 1991; 1994; Animalu and Santilli, 1995; Animalu *et al.*, 2009; Akpojotor, 2009; Akpojotor and Animalu, 2011) that the physical condition to trigger the hadronic bound state into the Cooper pairs and their propagation in the parent material for it to transit into the superconducting state is the effective valence z . The implication is that the g now depends on this z . Further, it is also argued here that w has been simulating the ionic radii I_r of the lattice so that the T_c is proportional to the I_r :

$$T_c \propto I_r \tag{13}$$

Therefore a T_c expression for the isosuperconductivity theory will depend on two factors, z and I_r . Since the BCS T_c expression in Eq. (11) also obviously depends on two factors, λ and w , which have been shown here to be simulating the z and I_r respectively, then re-expressing the BCS T_c expression to depend on the z and I_r is a natural starting point to formulate an isosuperconductivity T_c expression, that is,

$$T_c \approx \frac{\hbar I_r}{k_\beta} e^{-1/z} \tag{14}$$

Like the BCS T_c expression, an increase in either the z or I_r will increase the T_c value.

However, the BCS theory requires the λ to be small: it is widely believed that λ can take values from 0 – 1 so that its predicting power is limited to $T_c \leq 25$ K (Ginzburg and Kirzhnits, 1987) while the electron-phonon interaction will form polarons for $\lambda > 1$ (Alexandrov, 1996; Akpojotor and Oseji, 2002). In the isosuperconductivity theory, the z is not restricted to a small value because depending on the parent material, z ranges between 0 - 6 and consequently its predictive power can reach room temperature and even beyond. The implication is that the BCS T_c can simulate the transition temperature for the so-called weak-coupling superconductors since the exponential part of both Eqs. (11) and (14) will give the same result for λ and z ranging between 0 - 1 as shown in Table 1. Thereafter, it is only the isosuperconductivity T_c that can give higher T_c for $z > 1$. To depict the higher T_c for $z > 1$, a pre-exponential value of 68x which give a T_c ranging from 0 – 25 k for x ranging from 0 - 1 is used for $x = z$ ranging from 1 – 6 and the result is shown in Fig. 1. Observe that as expected, the highest critical temperature for $x = \lambda$ ranging from 0 – 1 is $T_c \sim 25$ K while the semi-empirical T_c obtained in this studies predicts higher T_c values even beyond room temperature (297 K).

The implication is that the isosuperconductivity T_c expression can be used to account for both low temperature superconductors such as metallic superconductors and their alloys, organics superconductors, heavy fermions as well as the relatively high T_c superconducting materials such as the cuprates and the iron based compounds. There is currently no simple method to determine the effective valence of the various superconducting materials since its value varies with the stoichiometry of the parent material. However, it is possible to make prediction using the knowledge of the valence range of the dominating element. For example, iron with a configuration of $3d^64s^2$ has an ionic configuration of either Fe^{2+} for $3d^6$ or Fe^{3+} for $3d^5$ (Akpojotor, 2009). Thus these values will give an idea of its effective valence when combined with other elements to form the superconducting parent material. The outlook here is to understand and determine the effective valence as well as the ionic radii and use them

as the control parameter for driving the superconductivity of the parent materials. The results obtained here can serve as a guide to obtain the values of the effective valence since from Fig. 1(a), the z of materials superconducting at $T_c < 25$ K will be from 0 – 1 while those at $T_c > 25$ K will have z values that are commensurate with their T_c values as predicted in Fig. 1(b). Thus a simple test of the isosuperconductivity theory will be to determine if it is able to predict the effective valence of superconducting materials even within the limit of the semi-empirical T_c expression obtained in this study and this is an open problem for both experimental chemists and physicists.

Table 1. The exponential part of the T_c expression (a) for both the electron-phonon coupling constant, l and the effective valence, z represented by x , that is, $x = z = l$ ranging from 0 to 1 and (b) only for z ranging from 1 to 6.

x	$T_c \approx e^{-1/x}$	z	$T_c \approx e^{-1/z}$
0.0	0.00000000	0.0	0.00000000
0.1	0.00004540	0.5	0.13533528
0.2	0.00673795	1.0	0.36787944
0.3	0.03567399	1.5	0.51341712
0.4	0.08208500	2.0	0.60653066
0.5	0.13533528	2.5	0.67032005
0.6	0.18887560	3.0	0.71653131
0.7	0.23965103	3.5	0.75147729
0.8	0.28650479	4.0	0.77880078
0.9	0.32919298	4.5	0.80073740
1.0	0.36787944	5.0	0.81873075
	(a)	5.5	0.83375292
		6.0	0.84648172
			(b)

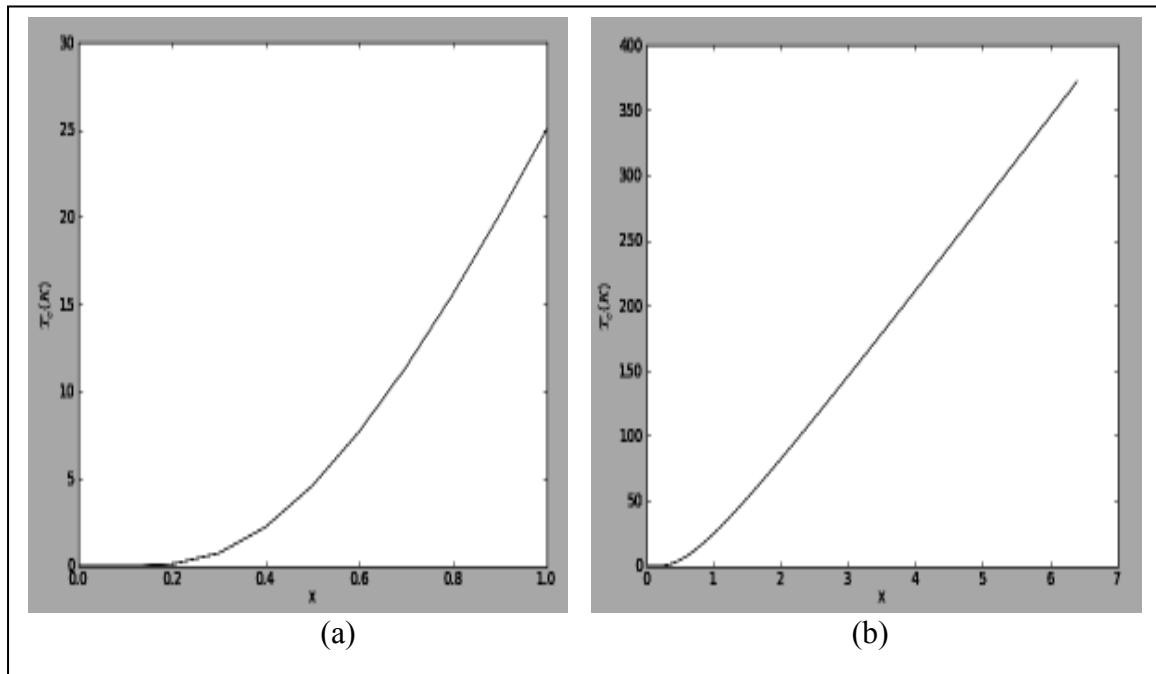


Fig 1: (Colour online) The critical temperature, T_c expression variation with both the electron-phonon coupling constant, l and the effective valence, z represented by x , that is, $x = z = l$ using a pre-exponential value of $68x$. The motivation for using this pre-exponential value is that it gives the values of the BCS T_c from 1 – 25 K for $l = 0 – 1$. Here it is the semi-empirical T_c expression that varies with (a) the $x = z = l$ ranging from 0 to 1 with the highest T_c at 25 K which is the highest predicted by the BCS theory because the l is restricted to small values ranging from 0 to 1 and (b) $x = z$ ranging from 1 to 6 with the highest T_c value beyond $T_c > 297$ K which is the room temperature.

SUMMARY AND CONCLUSION

The results presented in this work emanates from the consideration that at short distances, the effective valence will trigger the formation and propagation of the Cooper pairs as a result of an attractive Hulthen potential. At large distances (> 1 fm), however, the Hulthen potential no longer dominates and it has been

suggested by Akpojotor (2008) that the CPF is by superexchange interaction which naturally affects electrons that are close enough to have (no deep) overlapping wavefunctionis and this is purely a quantum mechanical affairs (Gandzha. and Kadeisvily, 2010). This dichotomy emanates from the regimes of validity of hadronic mechanic and quantum mechanics.

For sufficiently large distances, particles can be described effectively in point-like approximation so that there is the sole presence of action-at-a distance potential interaction which can be represented by a Hamiltonian. This approximate point-like description is no longer valid at sufficiently small distances so that their interaction is dominated by a contact non-potential character. As explained in (Gandzha. and Kadeisvily, 2010; Akpojotor and Animalu, 2011), the quantum mechanical point-like description is only not applicable in this regime since it is not violated because the condition in the regime was beyond what quantum mechanics was conceived for. The situation can be likened to the turn of the 20th century when many of the phenomena and experiments in physics at the macroscopic regime can conveniently be accounted for with the classical mechanics (Newton's laws, electromagnetism, optics and thermodynamics) but have very limited successes when applied to the new phenomena and experiments in physics at the microscopic scale. It was the boldness of Max Planck to successfully propose his revolutionary idea of quantization of energy and radiation to formulate the law for the microscopic phenomenon of blackbody radiation that gave birth to quantum physics. This boldness continued with the other founding fathers of this discipline with the mathematical formulation of the physics at the microscopic scale to give birth to quantum mechanics. From the Correspondence Principle, both classical mechanics and quantum mechanics are successful at the regimes of their validities, with the former being an approximation of the later which is more fundamental.

One of the early difficulties of quantum mechanics is that the behaviour of physical systems on the microscopic scale often seems peculiar, and the consequences of quantum theory are accordingly difficult to understand and to believe. Its concepts frequently conflict with common-sense notions derived from observations of the everyday world. There is no reason, however, why the behaviour of the microscopic world should conform to that of the familiar, large-scale world. It is important to realize that quantum mechanics is a branch of physics and that the business of

physics is to describe and account for the way the world on both the large and the small scale actually is and not how one imagines it or would like it to be. Ipso facto, it is pertinent to point out here that superconductivity is a microscopic phenomenon emanating from the interaction of particles with short distances beyond the point-like interaction of quantum mechanics and therefore requires to be investigated with the strong valence of the hadronic-type. This is the challenge and opportunity for the scientific world as we enter into the next century of the discovery of superconductivity.

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