ON THE OPTIMAL LOOK AT EXPRESS LINE PERFORMANCE`

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ABSTRACT

We examine a waiting process involving checkouts in superstores where time consciousness is seriously in practise in most competitive businesses. The paper proposes a technique for assessing the influence of express lines/ways on the waiting process. An optimization model is design to minimize the mean waiting time taking into cognisance the maximum number of items/ products permitted in the express line. The intent of the work is based on a real case of a do – it – yourself exclusive superstore, however the approach can be applied generally. A significant point of the paper is that the optimal operation of express lines does not improve the mean waiting time glaringly, however the influence of non-optimal operation can be very unfavourable.

Keywords arrival rate/ time, service time , waiting time, express lines/ways, queue models

INTRODUCTION

In a competitive world or market, business, entities must make their enclaves sufiscated in order to survive. This implies that to stay competitive in the global economy, a company must constantly strive to improve individual performance along the supply chain. This entails that organisations must to adopt various management practices that will reduce costs and enhance organisational effectiveness along the chain (Lo *et al.,* 2009). This attitude is spreading into specific factors such as time. Thus, as time-consciousness is becoming a relevant means of breaking even in competitive market, every business entity must strike and improve time related performance measures of service systems and this eventually becomes a major concern for management talk (De Toni and Menaghetti, 2000) This trend has made some supermarkets to advertise their products emphasising on short waiting time at checkout counters; even fast food restaurants and home delivering services offer consolation prizes if waiting time for an order exceeds certain limit time. Sherman (2010) opined that everyone has experienced waiting in line, whether at a fast-food restaurant or a supermarket, on the phone for technical assistance or at the Doctor's or the action Doctor's office or even the Bank. Futhermore, in each of these points, we meet with one form of experience, pleasant or unpleasant. Similarly Kostecki(1996) opined that this trend has made an important marketing and operational issues in several situations. For instance, the traditional supply chain management (SCM) approach of constructing operational strategies focuses on how to ensure reliable delivery with minimum operating costs and on maintaining a low inventory cost/buffer, by effectively managing the supply function on the upper stream along the entire chain of business units (Hoover et al., 2001).

One way of enhancing the waiting process in services is the application of line structuring rules. To fit into these rules, we need to understand how customers are categorized and directed to various checkouts. Customers are classified according to certain characteristics and different classes of customers are by demand directed to certain checkouts. One popular measure of line structuring rules is the application of express lines at checkouts of superstores or exclusive markets. Here customers are classified according to the number of items purchased and this attracts certain incentives or certain stipulated limits or value. One commonly used incentive is the use of the express lines. As another measure of enhancing waiting processes, Negroni (2012) examined how to trim the time spent in line next to one's travel time. In her argument, she said that more than 10 years after the September 11 terrorist attacks that completely altered the airport experience, travellers have a variety of options that will shorten waiting times at security and immigration. However, speedier processing has some downsides.

Business operators (mainly those in services with time-conscious competitive environs) who are goal-oriented considers customer satisfaction as a primary objective and waiting time an essential ingredient to success and so make considerable effort to maximize and minimize them respectively (Heskett *et al.,* 1994; Jones and Dent, 1994; Kostecki, 1996). A well known approach to waiting time reduction, dwell on queue theory. A queuing model is design to express the relation of service speed and waiting time. These models enable us to evaluate the sum of the cost of service and the cost of waiting (Bitran and Mondschein,1997). Hillier and Lieberman (1995) upon examining these models, saw that slow service is cheap but associated with high waiting cost, while fast service is associated with low waiting cost; thus a U-shaped curve describes the total cost of waiting. Since it is obvious that the exact value of the cost of waiting is not always given, minimization of the total cost is then the classical objective function of queuing problems.

Most managers in time –conscious competition have redress their attention to these features of the service process that have direct time consequences. As a merit, the classical cost immunization of waiting time, Hill *et al* (2002) opined that one new area of service process improvement is the search for the best configuration of lines for waiting time and reduction and for both service level. Lots of papers abound that studied the effects of changing a waiting process, some are simple case with queuing formulae that leads to quick and acceptable estimates of the performance measures. Amongst these are Andrews and Parsons (1989) and Erikar and Vinod (1989). However there are theoretical papers about performance of typical line built up. For example Shell and Babbar (1996) applied to compare waiting characteristics of four different basic service process design.

At times queuing formulae may provide inappropriate approximates of real life situation, in such situation, direct observation of the effect of process change is applied. For example Luo *et al.* (2004) studied the effect of express lunch service in a fast food restaurant. Results were gathered based on direct observation of the old and new service process. It was observed that if change to the service process is costly and management would prefer a priori estimate of the expected results, so simulation studies are applied. However simulation is expensive and time consuming, and the merit can be high if the appropriate service process is chosen. One frequently applied measure to improve the waiting process is the application of line structuring rules. Line structuring rules classify customers with respect to certain attributes and direct those different classes of customers to different services (Hillier Lieberman, 1995).

Several other works on selecting waiting line and what influence customer satisfaction may have on a waiting line and transform the actual waiting time into a preserved waiting time of the customer can be found (Whitt, 1999; Carmon *et al.,* 1995; Chebat *et al.,* 1995; Katz *et al.,* 1991; Niel, 2000). In spite of all these, in time –conscious business environs, the actual waiting time is an essential characteristic of the waiting process and this may be minimized directly or indirectly.

In view of the foregoing, the above, the intent of this paper is to study the effect or influence of express line. As a follow-up, a model is proposed to determine the optimal value of the parameter that controls the access to express lines. This control parameter (limit value) is defined as the maximum number of items purchased by customers in the express line and its optimal value minimizes the mean waiting time in all lines.

The scenario of the express line being presented here is based on a real life situation and has to do with a store considering the utilisation of express lines, but wanting to know the consequences of the new system. Thus the analyses of the lines in the store led to the formulation of a new optimization technique or model.

DETERMINATIOH OF THE MEAN WAITING TIME AS A FUNCTION OF THE LIMIT PARAMETER

Surely in this section we shall be examine/determine how the mean waiting time depends on the limit parameter. In doing so, we hope to consider two customers groups, as we view the valves of main queuing parameters such as arrival rate, mean service time, and variance of service times expressed as the attributes of all customers. We shall provide queuing models that are appropriate for constructing express line systems and formulae for locating mean waiting time.

NECESSARY EXPRESSIONS FOR THE VARIOUS PARAMETERS OF DIFFERENT CUSTOMER GROUPS

We denote the mean waiting time, which depends on the limit parameter as (tq) and (L) respectively. Thus the derivation of this function will make use of the notations in Table 1

Table 1 Notation

λ	arrival rate to the checkouts
λ_E	arrival rate to the express lines
λ_R	arrival rate to the regular lines
σ_E	standard deviation of the service time in the express lines
σ_R	standard deviation of the service time in the regular lines
t_i	mean service time of a customer buying i units of items
t_E	mean service time in the express lines
t_R	mean service time in the regular lines
t_qE	mean waiting time in the express lines
t_qR	mean waiting time in the regular lines
t_q	mean waiting time in all lines
p_i	probability of buying i units of items
a	fix time of payment
b	variable time of payment
H	mean number of items bought by customers
N	sample size
K	maximum unit of items bought by customers
L	maximum unit of items in the express lines (limit parameter)
R	number of regular lines
E	number of express lines
S	total number of lines

and bearing in mind the following conditions:

The arrival processes of the two customer groups will be described with the Poisson processes with arrival rates, λ_E and λ_R respectively. Recognizing the Renyi's limiting distribution theorem, the Poisson process is invariant for independent rare function and coordinate transformation. This implies that the distribution function of the time interval between consecutive events is the same, Renyi (1956), Szantai (1971). From the generalizations of this limiting distribution theorem see Szantai (1971); the arrival processes of the two customer groups can be approximated with the Poisson processes.

Customers arrive to the checkouts according to a Poisson process with arrival rate of λ .Customers buying L or fewer items join the express line. Maximum number of items brought by a customer is assumed to be K.

S checkouts are available each with their own lines. Along sides the S checkouts, are E express lines and R regular lines such that $S = E + R$. Distribution of customers is uniform among the same type of lines. Also distribution of customers among express E and regular R lines and can be determined by the numbers of items purchased. Thus of the probability of purchasing i units of items is p_i , then arrival rate to the express lines λ_E and to the regular lines λ_R are

$$\lambda_E = \lambda \sum_{i=1}^{L} p_i$$

and

$$\lambda_R = \lambda \sum_{i=L+1}^{k} p_i \qquad \text{(3.1)}$$

These two equations implicitly, assume that the number of customers that mandatorily join an express line, and customers that do not jostles between E and R lines.

Service time at the checkouts is a function of the number of items bought by the customers. This is assumed to be a linear function of number of items bought.

Let "b" be the slope of the regression line representing time in introducing the data of a single item into the cash register; the constant "a" of the regression line indicates the fixed time of payment, that is the number of items bought e.g. the mean time that a customer spends on paying his bill. The expected service time of customers buying i units of item (t_E) can be obtained from

$$t_i = a + b.i, i = 1, \cdots, k \qquad \text{(3.2)}$$

The mean service time of customers in the express line (t_E) is the weighted mean of the service time of customers buying L items or less. Similarly the mean service time of customers in the regular line (t_R) is the weighted mean of the service time of customers buying more than L items. The mean service time for each line can be expressed as

$$t_E = \frac{\sum_{i=1}^{L} p_i(a+b.i)}{\sum_{i=1}^{L} p_i}, \quad \text{and}$$

$$t_R = \frac{\sum_{i=L+1}^{k} p_i(a+b.i)}{\sum_{i=L+1}^{k} p_i} \qquad \text{(3.3)}$$

The variance of service time can be computed from two sources bearing in mind that service time is a function of the number of items purchased and so the general distribution will assume the knowledge of variance of service time of the express lines (σ_E^2) and that of regular lines (σ_R^2).

The two sources include;

Customers buying the same number of items may demand the same service time and so the variance of t will be (σ_E^2).

Checkout customers with different number of items in the basket will demand different service times; hence we calculate these variance for (σ_E^2) and (σ_R^2) using

$$\sigma_E^2 \approx \frac{\sum_{i=1}^{L}[p_i N.\sigma_{t_i}^2] + \sum_{i=1}^{L}[p_i N.(a+b.i - \sum_{i=1}^{L} p_i(a+b.i))^2]}{\sum_{i=1}^{L} p_i N - 1}$$

and

$$\sigma_R^2 \approx \frac{\sum_{i=L+1}^{K}[p_i N.\sigma_{t_i}^2] + \sum_{i=L+1}^{K}[p_i N.(a+b.i - \sum_{i=L+1}^{K} p_i(a+b.i))^2]}{\sum_{i=L+1}^{K} p_i N - 1} \quad 3.4)$$

where N is an assumed sample size. The mean waiting time in line (t_q) is the weighted mean of the mean waiting time in the express lines and regular lines. Thus we can compute the weights as proportion of customers in the two different line types a

$$t_q = t_q E \sum_{i=1}^{L} p_i + t_q R \sum_{i=L+1}^{K} p_i \quad \ldots\ldots\ldots\ldots(3.5)$$

HOW DO WE CHOOSE THE QUEUING MODEL?

Given S checkouts the express line system, with E lines and R regular lines; with the knowledge that each checkout has its own lines and the distribution of customers among the checkouts is even; one might think that S number of M|G|1 queuing systems (E for express lines and R Regular lines) can describe the performance of the operation of the checkout system. Unfortunately this is not so. We examine two possibilities of choosing our model as follows;

First, most customers arriving the checkout do not choose the lines randomly; within this scenario, some try to join the shortest line. Others try to estimate the workload (total service time) belonging to each line on arriving and pick the one with the smallest estimated workload. Secondly, some customers switch from slow moving lines to fast moving ones. In this process, some are likely to be idle; customers will still attempt to join those idle ones. A look at these cases, S (number of parallel working M|G|1 models) provides a worst case estimate of the operation of the checkout system. This choice of model assumes a random selection of lines and thus neglects the waiting time reduction effect of workload estimation and the switching of customers. A second approach of choosing a model is to model the checkout system as two M|G|K, queuing systems, with $K = E$ for the express lines and $K = R$ for regular lines. This approach is comparatively optimistic.

Consider first, that there are S different lines. If the length of the line varies distinctively, most customers tend to choose the lines randomly. Secondly, not all the customers will want to switch between slow moving and fast moving ones. As a result, if there emanate idle checkouts however, switching to these lines is difficult or switching does not save considerable time for the customer, they may decide to give up switching. In conclusion, the two M|G|K models provides a best case estimate of the operation of the checkout system and equally assumes an optimally efficient selection of lines and over look the effects of occasionally idle checkouts.

Now consider the M|G|1 and M|G|K approaches for the upper and lower estimates respectively of the waiting time in line. The mean waiting time is computed with different queuing models. To show which model is employed, the notation of the model is presented in angular brackets to $t_q, t_q E, and, t_q R$. Hence in worst case estimate, assume E independent express lines each with a λ_E / E arrival rate and R regular lines each with a λ_R / R arrival rate.

The following formula for the calculation of the mean waiting time in line, in an M|G|1 queue system was obtained through a combination of Pollarczek-Khuntchine formula and the use of equation (3.5).

$$t_q[M/G/1] = t_q E[M/G/1]\sum_{i=1}^{L} p_i + t_q R[M/G/1]\sum_{i=L+1}^{K} p_i \quad \ldots\ldots. (3.6)$$

On the best case scenario, two M|G|K systems are assumed with $K = E$ for the express lines and $K = R$ for the regular. Whit (1999) suggested a strategy for estimating t_q in an M|G|K system and through that we obtain the formula

$$t_q = [M/G/S] t_q E[M/G/E] = \frac{[1+\sigma_E^2]/t_E^2}{2} t_q E[M/M/E]\sum_{i=1}^{L} p_i + \frac{[1+\sigma_R^2]/t_R^2}{2} t_q R[M/M/R]\sum_{i=L+1}^{K} p_i \quad \ldots. (3.7)$$

In the light of this, our objective will be to minimize (3.6) and (3.7) with respect to L and then determine the optimal value of L numerically.

THE CHECKOUT SYSTEM AND STATISTICAL ANALYSIS OF THE PARAMETERS

Our analysis will dwell on Esco superstores that deal on do-it-yourself tools, household materials, equipments, etc and; relevant data used in this paper were slightly changed for confidential reasons; however the main conclusion reached are valid for the situation of the store.

We considered the statistical description of two processes: arrival and service; besides selecting a proper queuing model in analyzing queuing systems.

ANALYSIS OF THE ARRIVAL PROCESS.

This entails the arrival rate of customers to their checkouts. It is assumed that customers do not leave the store due to intolerable long lines at the checkouts and also, that cashiers served all customers duely during working hours. By this, the arrival rate of customers can be approximated by the number of customers served on a line unit. The checkout information system can easily provide this information and we observed that the hourly number of bills generated by the cashier can as well approximates the arrival rate properly. Though, this number varies within operating hours. Also, needed is a three dimensional database (hours, days, weeks) where these data are properly kept. With this database, periods with similar arrival rates are grouped together for the purpose of statistical analyses. For the purpose of statistical analyses of the database, one week was partitioned into one period with different arrival rates. Using the Poisson probability distribution on the Kolmogorov - Smirnov tests, we generate the following results in Table 2. For instance, 100 data are available for the morning periods (10.00 – 13.00) of four days (Mon - Thur) during 16weeks. Within these periods, the arrival rate is 90 customers per hour, and the significance of the Poisson distribution is (0.120).

Arrival rate of customers (arrival rate, significance level of Poisson distribution, samples)

We can see the Table of the analysis of the arrival process shows how the customers are distributed among the checkouts. Also the number of bills provided in a giving period by each cashier, indicates that their distributions among the checkouts are approximately uniform. The analysis of service process entails the statistical analysis of the service time of customers at checkouts. The service time often depend partly on the number of items purchased and partly on the mode of payment (credit card, exact exchange etc.). Data used here were collected not from checkout information system and the service time of customers was measured at different period per day and from the look of things, there is no significant different period. Thus statistical analysis provided a 1.21 minute mean service time.

Whereas in the express lines, customers joining the lines is a function of the number of customers directed to it and the service time and is determined by the number of items in their baskets. Hence this type of analysis demands information from also the statistical

	Mon Tue Wed Thu	Fri	Sat	Sunday
8.00 - 9.00	45(0.105) n = 60	35(0.303) n = 16	50(0.198) n = 16	Closed
9.00 - 10.00	90(0.155) n = 60	84(0.308) n = 11	160(0.398) n =14	154(0.377) n = 13
10.00 - 11. 12. - 13.00	110(0.098) n =150	120(0.116) n =83	182(0.207) n = 41	139(0.321) n = 21
13.00 - 14.00 14.00 - 15.00	90 (0.120) n = 100		153(0.168) n =51	
15.00 – 16.00 16.00 – 17.00	88(0.101)n = 110			121(0.177) n = 46
17.00 – 18.00 18.00 – 19.00	106(0.143)n = 110	126(0.165) n = 42	135(0.309) n =13	
19.00-20.00	104(0.209)n = 70		105(0.421) n =12	closed

properties of the number of items purchased by a customers and about the relationship between the number of items and the service time. From the CIS, the customers transactions can be located and thus an empirical density function of item per customer can be charted in figure B, indicating that most customers bought only very few items. Using Chi -square test, we were able to obtain a truncated geometric distribution which is acceptable with higher than 0.1, significance level and the mean number of items purchased by customers (b) as 3.089.

Similarly with service time of customers taken, the number of items belonging to each time period noted, and on the assumption of a linear relationship between numbers of item bought and service time, linear regression analysis was performed. The regression line can be seen in figure A, with slope (b =0.168minute)as mean reading time of a bar code of a single item. The constant of the regression line (a =0.513 minute) is the mean time that a customer spend on paying his bill. In the same vein, the correlation coefficient (0.893) arising from the analysis, the assumption of a linear relationship is acceptable.

CONCLUSION

In this paper, we have been able to provide analytical equations for the lower and upper estimation of the mean waiting time in the line with respect to the maximum number

Figure A

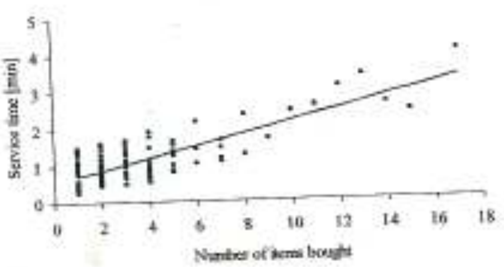
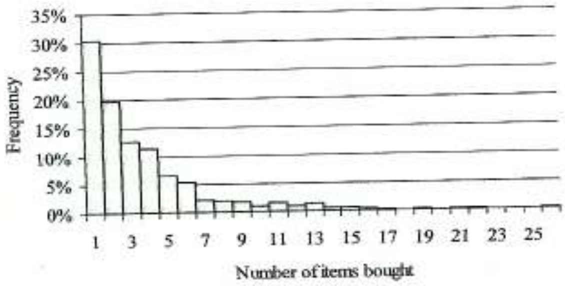


Figure B



of items allowed the express line. We were also able to establish that the mean number of items purchased per customer (H) is 3.089. From a linear regression analysis, an assumed linear relationship is acceptable, with a correlation coefficient (0.893). A more comprehensive decision could be reached if a numerical and sensitivity analysis is carried out on this study.

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